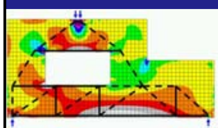
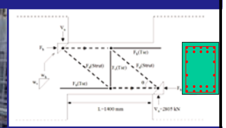
The Practicing Engineer's Guide to Designing by Strut and Tie Modeling (ACI 318-14)





presented by

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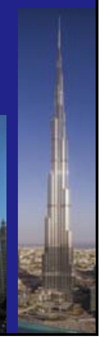
Outline

- *Behavior of Structures*
- *Code Requirements and Model Development*
- *Example / Summary*



Strut-and-Tie Methods

- *Tool for Design/Detailing of D-Regions*
- *Variable Angle Truss Analogy*
- *All Loading Conditions*
- *Several Solutions Exist for Any Problem*

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Strut-and-Tie Design Procedure

1. ***Define and Isolate D-Regions (from B-Regions)***
2. ***Compute Resultant Forces***
3. ***Devise Truss Model***
4. ***Calculate Truss Forces***
5. ***Check Struts and Nodal Zones ($\phi = 0.75$)***
   $F_{ns} = f_{ce} * A_{cs}$ where: $f_{ce} = 0.85 * \beta_s * f'_c$
   $\beta_s = 0.4, 0.8, 0.6, 0.75$ or 1.0
6. ***Provide Rebar for Ties ($\phi = 0.75$)***
   $F_{nt} = A_{ts} * f_y$

Basic Requirements

- *Model approximates stress flow*
- *Define component dimensions and strengths*
- *Define Phi and Beta Factors*
- *Analyze nodes and Anchorage*
- *Select Reinforcement Details*

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23.3.1 - STM Design Procedure

$$\phi F_{ns} \geq F_u$$

where

F_{us} = *Force in Strut/Tie/Node Due to Factored Loads*

F_{ns} = *Nominal Strength of Strut/Tie/Node*

Note: $\phi = 0.75$ *for STM (all failure modes)*

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23.4.1 – Strength of Struts

- Strut Without Longitudinal Reinf.

$$F_{ns} = f_{ce}A_{cs}$$

where

A_{cs} = Area at One End of Strut

f_{ce} = Smaller Effective Concrete Strength in Strut or Nodal Zone

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Strength of Struts

$$f_{ce} = 0.85\beta_s f'_c$$

• Prismatic Strut	$\beta_s = 1.0$
• Bottle-Shaped Strut	
- With Reinf. Per A.3.3	$\beta_s = 0.75$
- W/o Reinf. Per A.3.3	$\beta_s = 0.60\lambda$
• Strut in Tension Zone of a Member	$\beta_s = 0.40$
• All Others	$\beta_s = 0.60$

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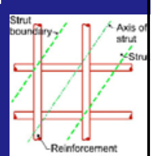
Reinforcement Crossing Struts

If $f'_c \le 6000$ psi

$$\Sigma\,[(A_{si} / b_s s_i)\sin(\alpha_i)] \ge 0.003$$

- A_{si} in Orthogonal Directions
- A_{si} in One Direction if $\alpha_i > 40°$



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Strength of Struts

- Strut With Longitudinal Reinf. Parallel to Strut Axis, and Enclosed in Ties or Spirals

$$F_{ns} = f_{ce}A_{cs} + A'_s f'_s$$

For Grades 40 to 60 Use $f'_s = f_y$

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23.7 – Strength of Ties

$$F_{nt} = A_{ts}f_y + A_{tp}(f_{se} + \Delta f_p)$$

where

- $(f_{se} + \Delta f_p) \le f_{py}$
- Bonded P/S — $\Delta f_p = 60$ ksi
- Unbonded P/S — $\Delta f_p = 10$ ksi
- A_{tp} is zero for nonprestressed members

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Strength of Ties

- Axis of Reinforcement to Coincide with Axis of Tie
- Proper Anchorage of Tie Reinforcement at Nodes
  - Mechanical Device
  - P/T Anchorage Device
  - Standard Hooks
  - Straight Bar Development

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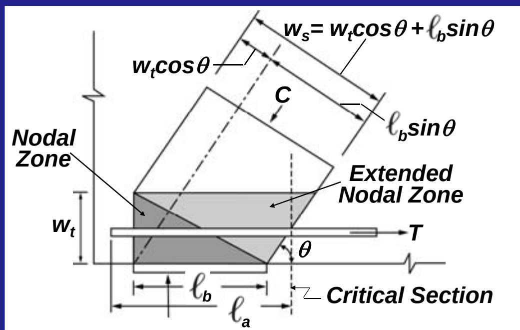
Development of Ties



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23.9 – Strength of Nodal Zones

$$F_{nn} = f_{ce}A_{nz}$$

where

f_{ce} = *Effective Concrete Compressive Strength in Nodal Zone*

A_{nz} = *Area of:*

- *Nodal Zone Face Perpendicular to* F_u
- *Section through Nodal Zone Perpendicular to Resultant Force on Section*

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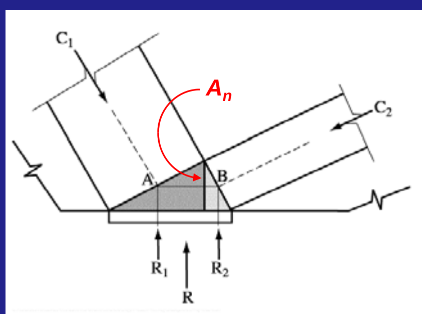
Area Through Nodal Zone



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Strength of Nodal Zones

$$f_{ce} = 0.85\beta_n f'_c$$

- C-C-C Node $\beta_n = 1.00$
- C-C-T Node $\beta_n = 0.80$
- C-T-T Node $\beta_n = 0.60$

(concrete good in compression, best in hydrostatic compression)

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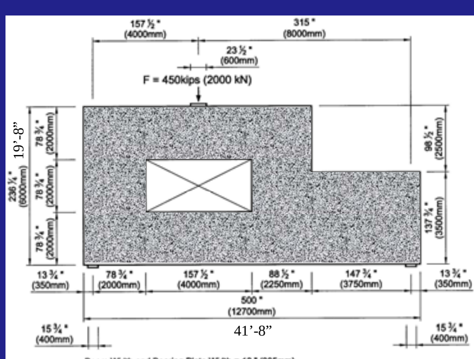
Example

System



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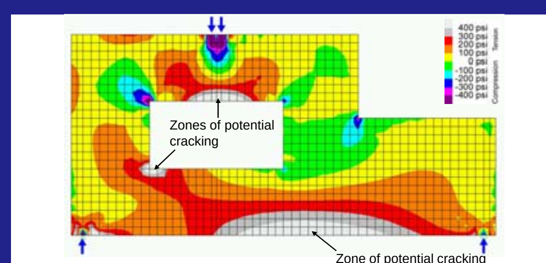
Strut-and-Tie Method: Finding the Model



- Display of maximum principal stress (ultimate load), undeformed shape

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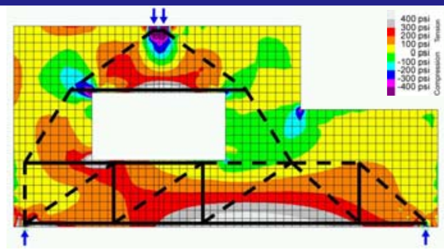
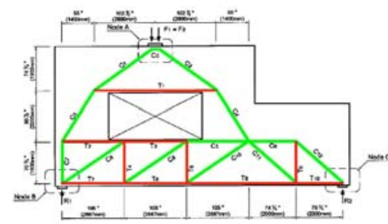
Strut-and-Tie Method: Finding the Model



• Display of maximum principal stress with the Strut-and-Tie Model (dashed lines are struts, solid lines are ties)

Strut-and-Tie Method: Analysis



• Geometry of the Strut-and-Tie Model (green lines are struts, red lines are ties)

3 Design Calculations

3.1 Check Bearing Strength

Note: the width of the bearing plate matches the beam width.

Strength Reduction Factor according to ACI 318-14 Chapter 21

$\phi = 0.75$

$\phi \cdot F_n \geq F_u$

$F_n = 0.85 \cdot f'_c \cdot A$

with

$f'_c = 4{,}500\,psi$

$\rightarrow F_n = 3{,}825\,psi \cdot A$

Plate at node A

$P = 450 kips$

$A = 23\tfrac{1}{2}'' \cdot 12'' = 282 in^2$

$\rightarrow 0.75 \cdot 3{,}825\,psi \cdot 282 in^2 = 809 kips \geq 450 kips \quad (3{,}598 kN \geq 2{,}000 kN) \rightarrow OK$

Plate at nodes B & C

$R_1 = 300 kips \geq 150 kips = R_2$

$A = 15\tfrac{3}{4}'' \cdot 12'' = 189 in^2$

$\rightarrow 0.75 \cdot 3{,}825 kips \cdot 189 in^2 = 542 kips \geq 300 kips \quad (2{,}410 kN \geq 1{,}333 kN) \rightarrow OK$

Note: The node checks will typically govern over the bearing checks

Design Calculations

Ties

$$\phi \cdot F_n \geq F_u \rightarrow F_n = \frac{F_u}{\phi}$$

with

$F_{nt} = A_{st} \cdot f_y + A_{ps} \cdot (f_{se} + \Delta f_p)$

$A_{ps} = 0 \text{ (no prestress)} \rightarrow F_{nt} = A_{st} \cdot f_y$

and

$f_y = 60{,}000\,psi = 60 ksi$

$$\rightarrow A_{st,req} = \frac{F_{nt}}{f_y} = \frac{F_u}{\phi \cdot f_y} = \frac{F_u}{0.75 \cdot 60 ksi} = \frac{F_u}{40 ksi} \qquad \left(\frac{F_u}{276 \frac{N}{mm^2}}\right)$$

The table shows the ties T_i with the forces, the required area of reinforcing steel, the bar size with the area of reinforcement for each bar, the number of bars, the provided area of reinforcement and the way the bars are distributed for each tie:

	F_{nt} [kips]	F_u [kN]	$A_{st,req}$ [in²]	Bar size	A_{st}/bar [in²]	No. of Bars	$A_{st,prov}$ [in²]	Distribution
T_1 =	167	744	3.72	#10	1.27	4	5.08	4#10
T_2 =	143	636	3.18	#8	0.79	4	3.16	4#8
T_3 =	34	151	0.75	#8	0.79	4	3.16	4#8
T_4 =	75	333	1.66	#4	0.20	10	2.00	10#4@18in
T_5 =	75	333	1.66	#4	0.20	10	2.00	10#4@18in
T_6 =	150	667	3.33	#4	0.20	18	3.60	18#4@9in
T_7 =	111	493	2.46	#10	1.27	6	7.62	6#10
T_8 =	222	987	4.93	#10	1.27	6	7.62	6#10
T_9 =	333	1480	7.40	#10	1.27	6	7.62	6#10
T_{10} =	167	741	3.70	#10	1.27	6	7.62	6#10

Design Calculations

Struts

Strength design

$\phi \cdot F_n \geq F_u$

With – nominal compressive strength of a strut without longitudinal reinforcement

$F_{ns} = f_{cu} \cdot A_c$

And for the effective compressive strength of the concrete in a strut

$f_{cu} = 0.85 \cdot \beta_s \cdot f'_c$

with

$f'_c = 4{,}500\,psi$

$\beta_s =$ 0.75 (with reinforcement satisfying 23.5 and normal weight concrete)

$\lambda = 1.0$ for normal weight concrete

$A_{c,req} = w_{s,req} \cdot 12''$

$\rightarrow f_{cu} = 2{,}869\,psi = 2.869 ksi$

$\rightarrow \phi \cdot F_n = 0.75 \cdot F_n = 0.75 \cdot 2.869 ksi \cdot A_c \geq F_u$

$$\rightarrow A_{c,req} \geq \frac{F_u}{0.75 \cdot 2.869 ksi} = \frac{F_u}{2.152 ksi} \qquad \left(\frac{F_u}{14.85 \frac{kN}{mm^2}}\right)$$

$$\rightarrow w_{s,req} = \frac{F_u}{2.152 ksi \cdot 12''} = \frac{F_u}{25.82 \frac{kips}{in}} \qquad \left(\frac{F_u}{4{,}525.9 \frac{kN}{mm}}\right)$$

Design Calculations

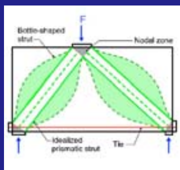
Struts

The table shows the struts C_i with the forces and the required and provided width of the strut with a 12" thickness. Where $w_{s,prov}$ is "ok", enough area is provided through the geometry of the model.

	F_u [kips]	F_u [kN]	d_{req} [in]	d_{prov} [in]
C_1 =	381	1695	15	ok
C_2 =	381	1695	15	ok
C_3 =	266	1185	10	ok
C_4 =	266	1185	10	ok
C_5 =	77	343	3	4", ok
C_6 =	167	741	6	ok
C_7 =	225	1000	9	24", ok
C_8 =	134	595	5	ok
C_9 =	134	595	5	ok
C_{10} =	134	595	5	ok
C_{11} =	224	997	9	ok
C_{12} =	224	997	9	ok

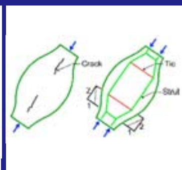
Design Calculations

Struts: Bottle-shaped Struts



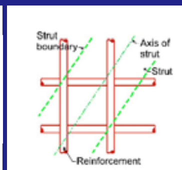
Actual Behavior of Struts in a Beam

- Idealized as an area with parallel borders
- Strut will form a bottle-like shape, if enough material to support this behavior surrounds the strut



Cracks in a bottle-shaped Strut

- Ties within the strut perpendicular to the axis of the strut
- Tension forces in the ties will try to create cracks at the axis of the strut



Reinforcing bottle-shaped Struts

- Reinforcement is required to resist cracking forces

Design Calculations

Nodes: General Notes

According to ACI 318-14 Section 29.3, the node at point A is a C-C-C-Node consisting of three struts, therefore b_n = 1.0, while the nodes at points B and C are C-C-T-Nodes, anchoring one tie each and therefore b_n = 0.8 for these nodes.

Nominal compression strength of a nodal zone

$$F_{nn} = f_{cu} \cdot A_n$$

with the calculated effective stress on a face of a nodal zone

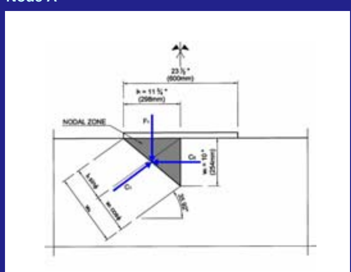
$$\phi \cdot f_{cu} = (0.75) \cdot 0.85 \cdot \beta_n \cdot f'_c$$

and

$$\phi \cdot F_n \geq F_u$$

Design Calculations

Node A



$F_1 = 225 kips$

$C_0 = 310 kips$

$C_1 = 383 kips$

$\theta = \arctan \frac{69.8}{96.4} = 35.92°$

$\beta_n = 1.0$

$w_{F_1} = l_b = 11\,3/4"$

$w_{C_0} = 10"$ (approximate height of compression block C_0)

$w_{C_1} = w_s = l_b \sin\theta + w_t \cos\theta = 15"$

$A_{F_1} = 11\,3/4" \cdot 12" = 141 in^2$

$A_{C_0} = 10" \cdot 12" = 120 in^2$

$A_{C_1} = 15" \cdot 10" = 150 in^2$

$F_{nn,F_1} = 0.85 \cdot 1.0 \cdot 4{,}500 psi \cdot 141 in^2 = 539 kips$ $(2{,}397 kN)$

$F_{nn,C_0} = 0.85 \cdot 1.0 \cdot 4{,}500 psi \cdot 120 in^2 = 459 kips$ $(2{,}042 kN)$

$F_{nn,C_1} = 0.85 \cdot 1.0 \cdot 4{,}500 psi \cdot 150 in^2 = 574 kips$ $(2{,}553 kN)$

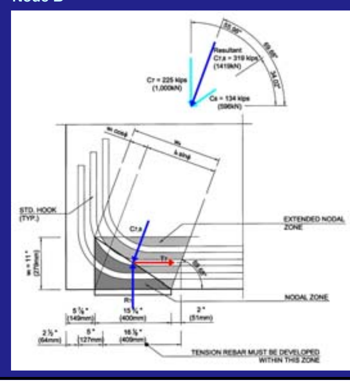
$0.75 \cdot 539 kips = 404 kips \geq 225 kips = F_1$ $(1{,}797 kN \geq 1{,}000 kN = F_1)$

$0.75 \cdot 459 kips = 344 kips \geq 310 kips = C_0$ $(1{,}530 kN \geq 1{,}379 kN = C_0)$

$0.75 \cdot 574 kips = 431 kips \geq 383 kips = C_1$ $(1{,}917 kN \geq 1{,}704 kN = C_1)$

Design Calculations

Node B



$R_1 = 300 kips$

$T_7 = 111 kips$

$C_{7,8} = 319 kips$

$\theta = 69.68°$

$\beta_n = 0.8$

$w_{R_1} = l_b = 15\,3/4"$

$w_{T_7} = w_t = 2 \cdot 2" + 3 \cdot 1\,1/4" + 2 \cdot 1\,1/2" = 10\,3/4"$

$w_{C_{7,8}} = w_s = l_b \sin\theta + w_t \cos\theta = 18\,1/2"$

$A_{R_1} = 15\,3/4" \cdot 12" = 189 in^2$

$A_{T_7} = 10\,3/4" \cdot 12" = 129 in^2$

$A_{C_{7,8}} = 18\,1/2" \cdot 12" = 222 in^2$

$F_{nn,R_1} = 0.85 \cdot 0.8 \cdot 4{,}500 psi \cdot 189 in^2 = 578 kips$ $(2{,}571 kN)$

$F_{nn,T_7} = 0.85 \cdot 0.8 \cdot 4{,}500 psi \cdot 129 in^2 = 395 kips$ $(1{,}757 kN)$

$F_{nn,C_{7,8}} = 0.85 \cdot 0.8 \cdot 4{,}500 psi \cdot 222 in^2 = 679 kips$ $(3{,}020 kN)$

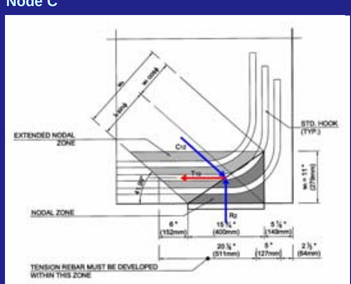
$0.75 \cdot 578 kips = 434 kips \geq 300 kips = R_1$ $(1{,}930 kN \geq 1{,}333 kN = R_1)$

$0.75 \cdot 395 kips = 296 kips \geq 111 kips = T_7$ $(1{,}317 kN \geq 494 kN = T_7)$

$0.75 \cdot 679 kips = 509 kips \geq 319 kips = C_{7,8}$ $(2{,}264 kN \geq 1{,}418 kN = C_{7,8})$

Design Calculations

Node C



$R_2 = 150 kips$

$T_{10} = 167 kips$

$C_{12} = 224 kips$

$\theta = 41.99°$

$\beta_n = 0.8$

$w_{R_2} = l_b = 15\,3/4"$

$w_{T_{10}} = w_t = 2 \cdot 2" + 3 \cdot 1\,1/4" + 2 \cdot 1\,1/2" = 10\,3/4"$

$w_{C_{12}} = w_s = l_b \sin\theta + w_t \cos\theta = 18\,1/2"$

$A_{R_2} = 15\,3/4 in \cdot 12 in = 189 in^2$

$A_{T_{10}} = 10\,3/4 in \cdot 12 in = 129 in^2$

$A_{C_{12}} = 18\,1/2 in \cdot 12 in = 222 in^2$

$F_{nn,R_2} = 0.85 \cdot 0.8 \cdot 4{,}500 psi \cdot 189 in^2 = 578 kips$ $(2{,}571 kN)$

$F_{nn,T_{10}} = 0.85 \cdot 0.8 \cdot 4{,}500 psi \cdot 129 in^2 = 395 kips$ $(1{,}757 kN)$

$F_{nn,C_{12}} = 0.85 \cdot 0.8 \cdot 4{,}500 psi \cdot 222 in^2 = 679 kips$ $(3{,}020 kN)$

$0.75 \cdot 578 kips = 434 kips \geq 150 kips = R_2$ $(1{,}930 kN \geq 667 kN = R_2)$

$0.75 \cdot 395 kips = 296 kips \geq 167 kips = T_{10}$ $(1{,}317 kN \geq 743 kN = C_{10})$

$0.75 \cdot 679 kips = 509 kips \geq 224 kips = C_{12}$ $(2{,}264 kN \geq 997 kN = C_{12})$

Design Calculations

Development Length

Node B governs as the available hook anchorage length is less than the one at node C.

The development length l_{dh} in a standard hook

$$l_{dh} = \frac{0.02 \cdot \beta \cdot \lambda \cdot f_y}{\sqrt{f_c'}} \cdot d_b$$

with

$\beta = 1.0$

$\lambda = 1.0$

$f_y = 60{,}000 psi$

$f'_c = 4{,}500 psi$

$d_b = 1.27"$ (#10)

$$\rightarrow l_{dh} = \frac{0.02 \cdot 1.0 \cdot 1.0 \cdot 60{,}000 psi}{\sqrt{4{,}500 psi}} \cdot 1.27" = 22\,3/4" \quad (577 mm)$$

Following ACI 318-14, which requires for a cover beyond the hook of less than 2.5” a spacing of stirrups not greater than 3d_b along $l_{dh,}$, and the first stirrup being within 2d_b of the outside of the bent, l_{dh} may be multiplied by 0.7 accordingly and therefore is reduced to

$$l_{dh} = 0.7 \cdot 22\,3/4" = 16" \quad (404 mm)$$

Design Calculations

Development Length

The anchorage length is sufficient for both nodes with l_a being

$$l_{a,NodeB} = \frac{\frac{w_c}{2}}{\tan 69.68^\circ} + 15\tfrac{3}{4}" + 5\tfrac{7}{8}" - 2\tfrac{1}{2}" - 5" = 16\tfrac{1}{8}" \qquad (409mm)$$

$$l_{a,NodeC} = \frac{\frac{w_c}{2}}{\tan 41.99^\circ} + 15\tfrac{3}{4}" + 5\tfrac{7}{8}" - 2\tfrac{1}{2}" - 5" = 20\tfrac{1}{8}" \qquad (510mm)$$

$$l_{dh} = 16" \leq 16\tfrac{1}{8}" \qquad (404mm \leq 409mm)$$

$\rightarrow$ Node B and Node C are acceptable

l_{dh} could be further reduced by the ratio of $A_{s,req}$ to $A_{s,prov}$ (3.7/7.62 = 0.49) additional reduction in l_{dh} has not been taken as it will not change the results.

Design Calculations

Minimum Reinforcement

The ratio between the area of reinforcement to gross concrete area is

$$\rho = \frac{A_S}{A_C} = 0.0025$$

Assume a reinforcement of #4 at 12", with $A_{st} = 0.20in^2$, therefore with the width of the wall being 12":

$$\rho = \frac{2 \cdot 0.20in^2}{12" \cdot 12"} = 0.0028 \geq 0.0025 \rightarrow \text{OK}$$

Conclusion: provide #4 at 12" minimum each way each face.

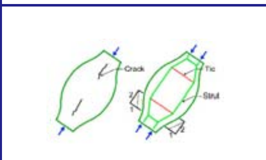
Design Calculations

Minimum Skin Reinforcement

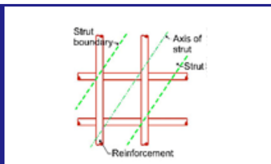
Checking the requirements, no additional the horizontal and vertical minimum reinforcement for shrinkage and temperature need be provided. If the effective depth of a beam exceeds 36", skin reinforcement shall be distributed according to ACI 318-14. The skin reinforcement is to be distributed for a distance of $d/2$ at a spacing not exceeding the least of $s_{sk} > d/6$, $s_{sk} > 12"$ and $s_{sk} > 1000A_b/(d - 30)$:

$$\frac{1000 \cdot 0.31in^2}{70\tfrac{1}{4}" - 30"} = 7\tfrac{1}{2}" \rightarrow s_{sk} \geq 7\tfrac{1}{2}" \qquad (193mm)$$

Therefore the horizontal reinforcement of #4 at 12" shall be changed to #5 at 7½" for the bottom of the deep beam and above the opening in a depth of 37".

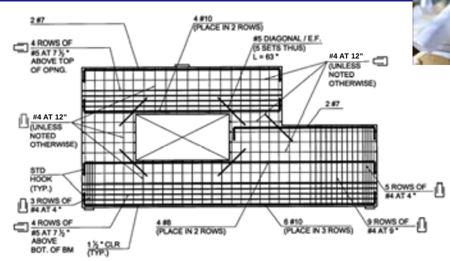


Cracks in a bottle-shaped Strut



Reinforcing bottle-shaped Struts

Reinforcing Bar Layout



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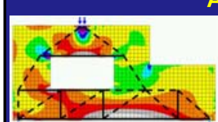
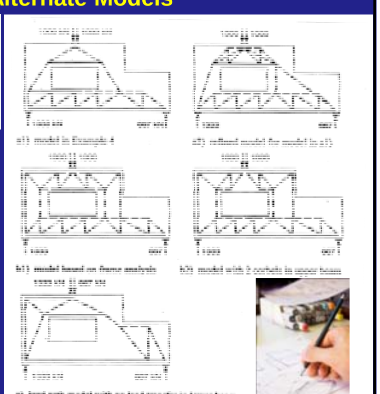
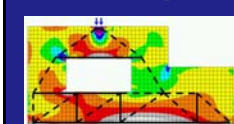
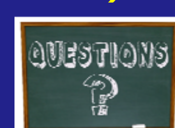
Alternate Models





The Practicing Engineer's Guide to Designing by Strut and Tie Modeling (ACI 318-14)





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